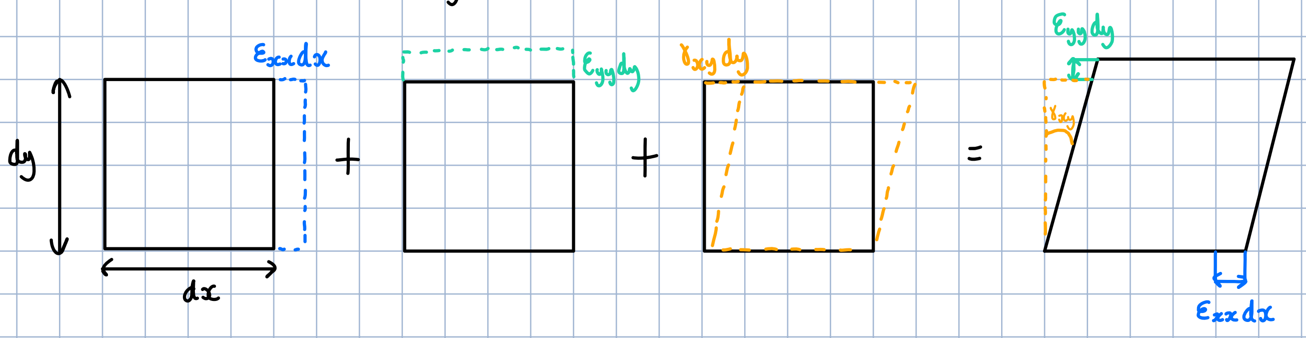
General 2D Strain : (exaggerated)



Plane Strain : deformations limited to XY Plane

→ Plane strain ≠ Plane Stress

for plane strain, $\varepsilon_{zz} = \gamma_{xz} = \gamma_{yz} = 0$ but $\sigma_{zz} \neq 0$

to achieve 0 strain in z, likely to be stress in z

for plane stress, $\sigma_{zz} = \tau_{xz} = \tau_{yz} = 0$ but $\varepsilon_{zz} \neq 0$

are cases when $\varepsilon_{zz} = 0$

Plane strain often thick structures

Plane stress often thin walled structures

although $\sigma_{zz} = 0$, thickness may vary due to Poisson's Ratio, ν

Plane stress results in $\varepsilon_{zz} = 0$ when :

→ Pure shear

→ or when $\sigma_{xx} = -\sigma_{yy}$: balance

tension reduces thickness, t

compression increases t

- Out-of-plane strains do not affect XY strain transformations

→ identical transformations for plane stress & strain :

$$\text{stress tensor } \bar{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{bmatrix}$$

where mathematical shear strain $\varepsilon_{xy} = \gamma_{xy}/2$

Strain Transformation Equations :

$$\begin{bmatrix} \varepsilon_{x'x'} \\ \varepsilon_{y'y'} \\ \gamma_{x'y'}/2 \end{bmatrix} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy}/2 \end{bmatrix}$$

$\left(\varepsilon_{xy} = \gamma_{x'y'}/2 \right)$

→ very similar to stress transformation

Principal Directions & Strains and Max/min shear strain :

- Replace σ_{xx}, σ_{yy} & τ_{xy} with ε_{xx}, ε_{yy} & $\gamma_{xy}/2$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_{xx} - \varepsilon_{yy}}$$

$$\varepsilon_{1,2} = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} \pm \sqrt{\left(\frac{\varepsilon_{xx} - \varepsilon_{yy}}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{max,min} = \pm \sqrt{(\varepsilon_{xx} - \varepsilon_{yy})^2 + \gamma_{xy}^2} = \varepsilon_1 - \varepsilon_2$$

⟶ Mohr's circle can be used for graphical approach :

↳ plot ε against $\gamma/2$

Experimental Methods for Strain Calculation / Analysis :

Discrete Methods :

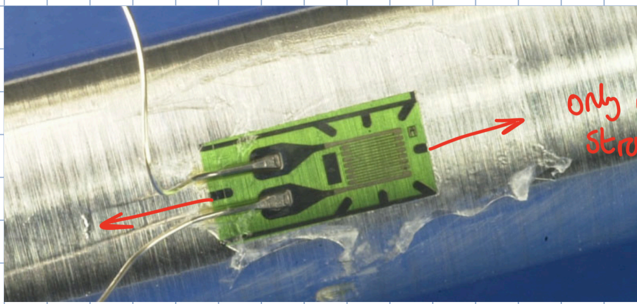
- Strain gauges

Full field Methods :

- Photo elasticity
- Digital Image Correlation (DIC)

→ experiment provides validation as there may be uncertainty in material properties or loads.

Strain Gauges:



only measures strains in this direction

Resistance, R, changes with strain (gauge factor K)

$$\boxed{\frac{\Delta R}{R} = K\varepsilon}$$

transverse & shear strain not measured

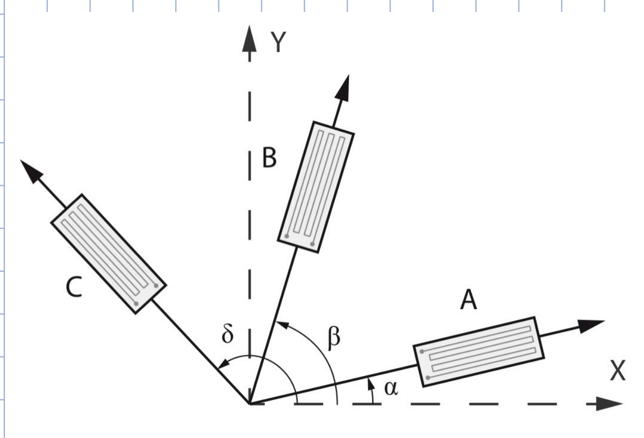
to reconstruct state of plane stress at a point, 3 independent strain measurements taken at different angles

↳ strain gauge rosettes

e.g. 0°/45°/90° 0°/60°/120°

Calculating Original Strain in XY System:

- Use one of the strain transformation equations & sub rosette strains at their angle:



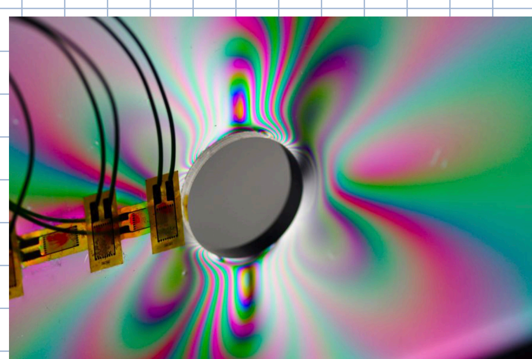
$$\varepsilon_A = \varepsilon_{xx}\cos^2\alpha + \varepsilon_{yy}\sin^2\alpha + \gamma_{xy}\sin\alpha\cos\alpha$$

$$\varepsilon_B = \varepsilon_{xx}\cos^2\beta + \varepsilon_{yy}\sin^2\beta + \gamma_{xy}\sin\beta\cos\beta$$

$$\varepsilon_C = \varepsilon_{xx}\cos^2\delta + \varepsilon_{yy}\sin^2\delta + \gamma_{xy}\sin\delta\cos\delta$$

- Solve simultaneously to find ε_{xx}, ε_{yy} & γ_{xy}.

Photo Elasticity:



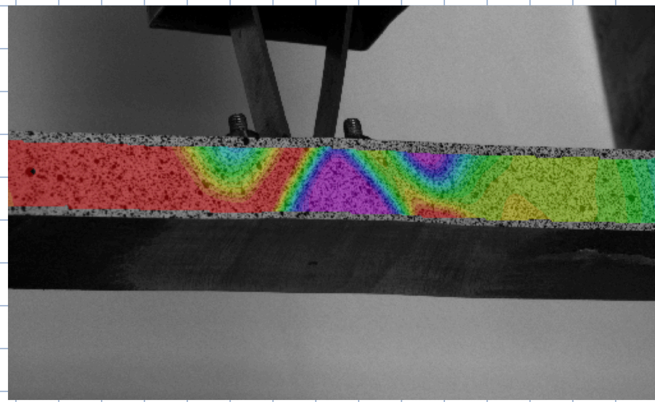
- Full-field measurement
- Relies on change of refractive index of coating with stress

→ isostatic lines show difference in principal stress $(\sigma_1 - \sigma_2)$
→ contours have constant shear stress
→ don't know magnitude of principle stress

Digital Image Correlation :

- tracks relative displacement (u,v) of fine speckled pattern

→ computationally intensive



- Defining strain equations via mathematical strain yields same results as earlier derivation.